M31

Coimisiún na Scrúduithe Stáit
State Examinations Commission

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Marking Scheme	*Leaving Certificate Examination, 2004*
Applied Mathematics	*Ordinary Level*

General Guidelines

1. Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

   Serious blunder or omission or misreading which oversimplifies:
   - award the attempt mark only.

   Attempt marks are awarded as follows: 5 (att 2), 10 (att 3).

2. Mark all answers, including excess answers and repeated answers whether cancelled or not, and award the marks for the best answers.

3. Mark scripts in red unless candidate uses red. If a candidate uses red, mark the script in blue or black.

4. Number the grid on each script 1 to 9 in numerical order, not the order of answering.

5. Scrutinise **all** pages of the answer book.

6. The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

1. Three points a, b and c, lie on a straight level road such that $|ab| = |bc| = 100$ m. A car, travelling with uniform retardation, passes point a with a speed of 20 m/s and passes point b with a speed of 15 m/s.

(i) Find the uniform retardation of the car.

(ii) Find the time it takes the car to travel from a to b, giving your answer as a fraction.

(iii) Find the speed of the car as it passes c, giving your answer in the form $p\sqrt{q}$, where $p, q \in \mathbf{N}$.

(iv) How much further, after passing c, will the car travel before coming to rest? Give your answer to the nearest metre.

(i)

$$v^2 = u^2 + 2as$$

$$15^2 = 20^2 + 2a(100) \quad 10$$

$$a = \frac{-175}{200} \text{ or } \frac{-7}{8} \text{ or } -0.875 \quad 5$$

(ii)

$$v = u + at$$

$$15 = 20 + \left(\frac{-7}{8}\right)t \quad 10$$

$$t = \frac{40}{7} \quad 5$$

(iii) stage bc:

$$v^2 = u^2 + 2as$$

$$v^2 = 15^2 + 2\left(\frac{-7}{8}\right)(100)$$

$$= 50$$

$$v = 5\sqrt{2} \quad 10$$

(iv) final stage:

$$v^2 = u^2 + 2as$$

$$0 = 50 + 2\left(\frac{-7}{8}\right)s$$

$$s = \frac{200}{7} = 28.57$$

$$s = 29 \quad 10$$

50

2. **(a)** Ship A is travelling due north with a constant speed of 15 km/hr.
Ship B is travelling north-west with a constant speed of $15\sqrt{2}$ km/hr.

(i) Write down the velocity of ship A and the velocity of ship B, in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find the velocity of ship A relative to ship B.

(iii) If ship A is 5.5 km due west of ship B at noon, at what time will ship A intercept ship B?

(b) Car P and car Q are travelling eastwards on a straight level road.
P has a constant speed of 20 m/s and Q has a constant speed of 10 m/s.

(i) Find the velocity of P relative to Q.

(ii) At a certain instant car P is 100 m behind car Q.
Find the distance between the two cars 3.5 seconds later.

(a) (i)	$V_A = 0\,\vec{i} + 15\,\vec{j}$	5
	$V_B = -15\sqrt{2}\cos 45\ \vec{i} + 15\sqrt{2}\sin 45\ \vec{j}$	5
	$= -15\,\vec{i} + 15\,\vec{j}$	
(ii)	$V_{AB} = V_A - V_B$	5
	$= (0\,\vec{i} + 15\,\vec{j}) - (-15\,\vec{i} + 15\,\vec{j})$	
	$= 15\,\vec{i} + 0\,\vec{j}$	5
(iii)	$\text{time} = \frac{5.5}{15}$	5
	$= 0.366$ hr	
	$= 22$ minutes	5
	$\text{time} = 12:22$	
(b) (i)	$V_{PQ} = V_P - V_Q$	5
	$= (20\,\vec{i}) - (10\,\vec{i})$	
	$= 10\,\vec{i} + 0\,\vec{j}$	5
(ii)	$\text{distance} = 100 + S_Q - S_P$	5
	$= 100 + 10(3.5) - 20(3.5)$	
	$= 65$ m	5

50

3 **(a)** A smooth rectangular box is fixed to the horizontal ground.
A ball is moving with constant speed u m/s on the top of the box.
The ball is moving parallel to a side of the box.
The ball rolls a distance 2 m in a time of 0.5 seconds before falling over an edge of the box.

(i) Find the value of u.

(ii) The ball strikes the horizontal ground at a distance of $\frac{4}{\sqrt{5}}$ m from the bottom of the box.
Find the height of the box.

(b) A golf ball is struck from a point r on the horizontal ground with a speed of 20 m/s at an angle θ to the horizontal ground. After $2\sqrt{2}$ seconds, the ball strikes the ground at a point which is a horizontal distance of 40 m from r.

(i) Find the initial velocity of the ball, in terms of $\vec{i}$ and $\vec{j}$ and θ.

(ii) Find the angle θ.

(a) (i)

$$s = ut + \tfrac{1}{2}at^2$$
$$2 = u(0.5) + 0$$
$$u = 4$$

10

(ii)

$$r_i = u(t)$$
$$\frac{4}{\sqrt{5}} = 4t$$
$$t = \frac{1}{\sqrt{5}}$$

10

$$r_j = 0 + \tfrac{1}{2}at^2$$
$$h = 0 + \tfrac{1}{2}(10)\tfrac{1}{5}$$
$$= 1$$

10

(b) (i) initial velocity $= 20\cos\theta\ \vec{i} + 20\sin\theta\ \vec{j}$

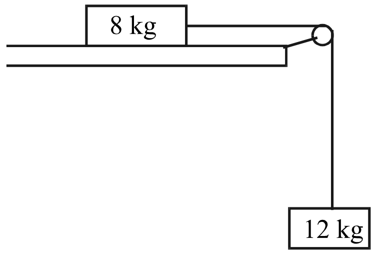
10

(ii)

$$r_i = 40$$
$$20\cos\theta.\left(2\sqrt{2}\right) = 40$$
$$\cos\theta. = \frac{40}{40\sqrt{2}} = \frac{1}{\sqrt{2}}$$
$$\theta = 45°$$

10 | 50

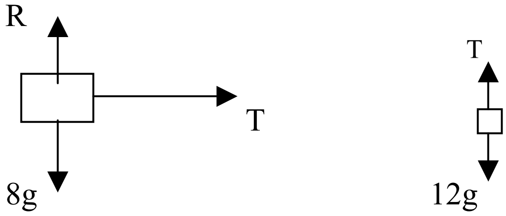
4. **(a)** Two particles, of masses 8 kg and 12 kg, are connected by a light, taut, inextensible string passing over a smooth light pulley at the edge of a smooth horizontal table.



The 12 kg mass hangs freely under gravity.
The particles are released from rest.
The 12 kg mass moves vertically downwards.

(i) Show on separate diagrams all the forces acting on each particle.
(ii) Find the acceleration of the 12 kg mass.
(iii) Find the tension in the string.

(a) **(i)**



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(ii)

$$T = 8f$$ 5

$$12g - T = 12f$$ 5

$$12g = 20f$$

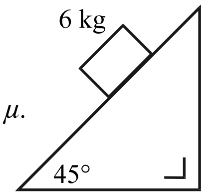
$$f = 6$$ 5

(iii)

$$T = 8f$$
$$= 8(6)$$
$$= 48$$ 10

30

(b) A particle of mass 6 kg is placed on a rough plane inclined at an angle of 45° to the horizontal. The coefficient of friction between the particle and the plane is μ. The particle is released from rest and takes 4 seconds to move a distance of $10\sqrt{2}$ metres down the plane.

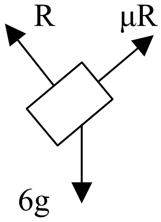


(i) Show on a diagram all the forces acting on the particle.

(ii) Show that the acceleration of the particle is $\frac{5\sqrt{2}}{4}$ m/s^2.

(ii) Find the value of μ.

(b) **(i)**



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(ii)

$$s = ut + \tfrac{1}{2}ft^2$$

$$10\sqrt{2} = 0 + \tfrac{1}{2}f(4)^2$$

$$f = \frac{10\sqrt{2}}{8} = \frac{5\sqrt{2}}{4}$$

5

(iii)

$$6g\cos 45 - \mu R = 6f$$

5

$$6g\frac{1}{\sqrt{2}} - \mu(6g\cos 45) = 6\left(\frac{5\sqrt{2}}{4}\right)$$

$$6g\frac{1}{\sqrt{2}} - \mu\left(6g\frac{1}{\sqrt{2}}\right) = 6\left(\frac{5\sqrt{2}}{4}\right)$$

$$60 - 60\mu = 15$$

$$\mu = \frac{45}{60} \quad \text{or} \quad \frac{3}{4}$$

5

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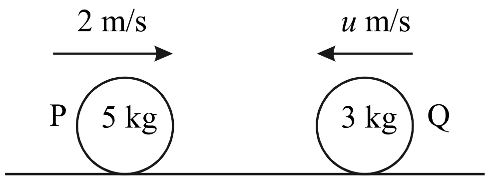
5. **(a)** A smooth sphere P, of mass 5 kg, moving with a speed of 2 m/s collides directly with a smooth sphere Q, of mass 3 kg, moving in the opposite direction with a speed of u m/s on a smooth horizontal table.



The coefficient of restitution for the collision is $\frac{1}{2}$.

As a result of the collision, sphere P is brought to rest.

(i) Find the value of u.

(ii) Find the speed of Q after the collision.

(b) A ball is dropped from rest from a height of 1.25 m onto a smooth horizontal table. The ball hits the table with a speed of v m/s and then rebounds to a height of h metres above the table.
The coefficient of restitution between the ball and the table is 0.8.

(i) Find the value of v.

(ii) Find the value of h.

(a) (i) PCM $5(2)+3(-u) = 5v_1 + 3v_2$

$10-3u = 5(0)+3v_2$ — 10

NEL $v_1 - v_2 = -e(u_1 - u_2)$

$0 - v_2 = -\frac{1}{2}(2+u)$ — 10

$u = \frac{14}{9}$ — 5

(ii) $v_2 = \frac{1}{2}(2+u)$

$= \frac{16}{9}$ — 5

(b) (i) $v^2 = u^2 + 2as$

$= 0 + 2(10)(1.25)$

$= 25$

$\Rightarrow \quad v = 5$ — 10

(ii) rebound velocity $= ev = (0.8)5 = 4$ — 5

$v^2 = u^2 + 2as$

$0 = 4^2 + 2(-10)h$

$\Rightarrow \quad h = 0.8$ — 5

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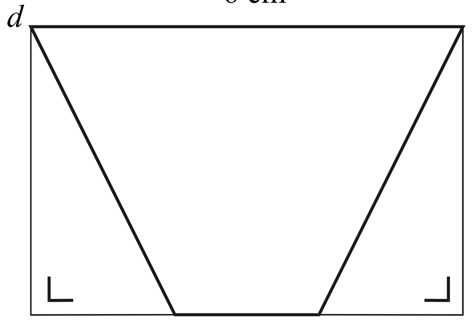
6. **(a)** A rectangular lamina *pqcd* measures 6 cm by 4 cm.
Two triangular pieces *dpa* and *cbq* are removed from the rectangular lamina to form the shape *abcd* as shown where $|pa| = |ab| = |bq| = 2$ cm.

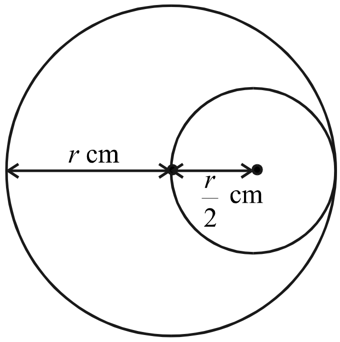


Find the distance of the centre of gravity of the shape *abcd* from $[ab]$.

(b) A uniform lamina is in the form of a circle of radius *r*.

A circle of radius $\frac{r}{2}$ is cut from the lamina.

The distance between the centres of the two circles is $\frac{r}{2}$.



Find the position of the centre of gravity of the remainder in terms of *r*, with respect to the centre of the circle of radius *r*.

(a)

$$\text{area of } dpa = \text{area } cbq = \tfrac{1}{2}(4)(2) = 4$$
$$\text{area } abcd = 6(4) - 4 - 4 = 16$$
10

$$24(2) = 4\left(\frac{4}{3}\right) + 4\left(\frac{4}{3}\right) + 16(\bar{y})$$
10

$$\bar{y} = \frac{7}{3}$$
5

(b)

$$\text{area of remainder} = \pi r^2 - \pi\left(\frac{r}{2}\right)^2$$
$$= \frac{3\pi r^2}{4}$$
10

$$\pi r^2(0) = \frac{3\pi r^2}{4}(\bar{x}) + \frac{\pi r^2}{4}\left(\frac{r}{2}\right)$$
10

$$\bar{x} = \frac{-r}{6} \text{ and } \bar{y} = 0$$
5

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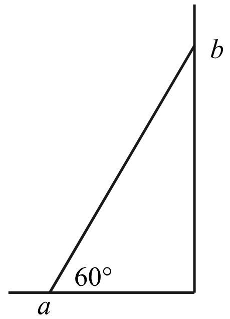
7. A uniform ladder, [*ab*], of weight W and of length 10 m, stands with end *a* on a rough horizontal floor and end *b* against a smooth vertical wall.
The coefficient of friction between the ladder and the ground is μ. The ladder makes an angle of 60° with the floor, as shown.



A man, whose weight is twice that of the ladder, climbs to the top of the ladder.

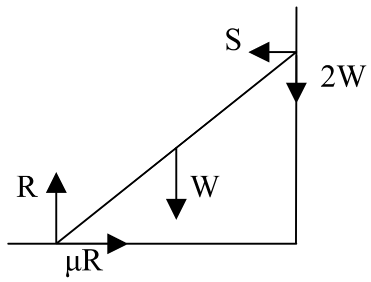
(i) Show on a diagram all the forces acting on the ladder.

(ii) Write down the two equations that arise from resolving the forces horizontally and vertically.

(iii) Write down the equation that arises from taking moments about the point *b*.

(iv) If the ladder is on the point of slipping, find the value of μ.

(i)



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(ii) horiz $S = \mu R$ 5

vert $R = 3W$ 5

(iii) Moments about b:

$$R\,(10\cos 60) = \mu R\,(10\sin 60) + W(5\cos 60)$$ 15

(iv)

$$R\,(10\cos 60) = \mu R\,(10\sin 60) + W(5\cos 60)$$

$$3W = 3\mu W \tan 60 + \frac{W}{2}$$

$$\frac{5W}{2} = 3\mu W\sqrt{3}$$

$$\mu = \frac{5}{6\sqrt{3}} \text{ or } 0.48$$ 15

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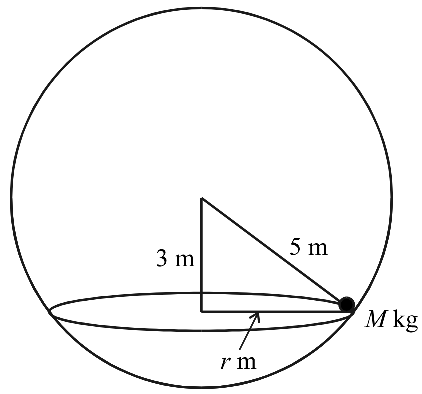
8. **(a)** A boy ties a 1 kg mass to the end of a piece of string 50 cm in length.

He then rotates the mass on a smooth horizontal table, so that it describes a horizontal circle whose centre is also on the table.

If the string breaks when the tension in the string exceeds 8 Newtons, what is the greatest speed with which the boy can rotate the mass?

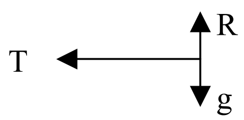
(b) A circus act uses a fixed spherical bowl of inner radius 5 m.
A girl and her motorcycle together have a mass of *M* kg, as shown in the diagram.
The girl and her motorcycle describe a horizontal circle of radius *r* m, with angular velocity *ω* rad/s, on the inside rough surface of the bowl.
The centre of the horizontal circle is 3 m vertically below the centre of the bowl.



The coefficient of friction between the motorcycle tyres and the bowl is $\frac{3}{4}$.

(i) Find the value of *r*.
(ii) Show on a diagram all the forces acting on the mass *M*.
(iii) Find the value of *ω*, correct to two decimal places.

(a)



$$T = \frac{mv^2}{r}$$

$$8 = \frac{1v^2}{0.5}$$

$$\Rightarrow v = 2 \text{ m/s}$$

10

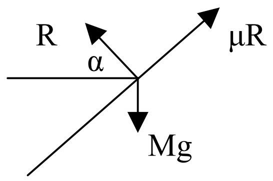
10 20

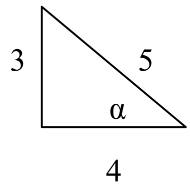
(b) (i)

$$r = \sqrt{5^2 - 3^2}$$
$$= 4$$

10

(ii)





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(iii)

$$R\sin\alpha + \mu R\cos\alpha = Mg$$
$$R\left(\frac{3}{5}\right) + \left(\frac{3}{4}\right)R\left(\frac{4}{5}\right) = Mg$$
$$\left(\frac{6}{5}\right)R = Mg$$

5

$$R\cos\alpha - \mu R\sin\alpha = Mr\omega^2$$
$$R\left(\frac{4}{5}\right) - \left(\frac{3}{4}\right)R\left(\frac{3}{5}\right) = M(4)\omega^2$$
$$\left(\frac{7}{20}\right)R = M(4)\omega^2$$

5

$$\left(\frac{7}{20}\right)\left(\frac{5}{6}\right)Mg = M(4)\omega^2$$
$$\omega^2 = \frac{70}{96} = 0.73$$
$$\omega = 0.85$$

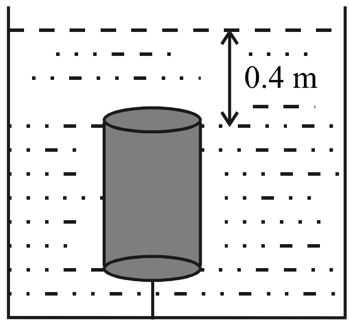
5 | 30

9. **(i)** State the Principle of Archimedes.

(ii) Calculate the pressure at a point in a liquid, of relative density 1.2, if the point is 0.4 m vertically below the surface.

A right circular solid cylinder has a height of 0.6 m and radius 0.2 m. The cylinder is held immersed in a tank of liquid of relative density 1.2 by a light inelastic string tied to the cylinder and to the bottom of the tank.

The top of the cylinder is horizontal and is 0.4 m below the surface of the liquid.



(iii) Find, in terms of π, the thrust downwards on the top of the cylinder.

(iv) Find, in terms of π, the thrust upwards on the bottom of the cylinder.

(v) Show that these results are in agreement with the Principle of Archimedes.

[Density of water = 1000 kg/m^3.]

(i) : Principal of Archimedes — 10

(ii) Pressure $= \rho g h$

$= 1200(10)(0.4)$

$= 4800$ — 10

(iii) Thrust = Pressure x Area

$= 4800\{\pi(0.2)^2\} = 192\pi$ — 10

(iv) Thrust = Pressure x Area

$= \{1200(10)(1)\}\{\pi(0.2)^2\} = 480\pi$ — 10

(v) $B = \rho V g$

$= 1200\{\pi(0.2)^2(0.6)\}\{10\} = 288\pi$

$480\pi - 192\pi = 288\pi$

$\Rightarrow$ these results are in agreement with the principle of Archimedes — 10

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